

# MANFIS Based Modeling and Prediction of the Driver-Vehicle Unit Behavior in Overtaking Scenarios

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## Abstract

Overtaking a slow lead vehicle is a complex maneuver because of the variety of overtaking conditions and driver behavior. In this study, two novel prediction models for overtaking behavior are proposed. These models are derived based on multi-input multi-output adaptive neuro-fuzzy inference system (MANFIS). They are validated at microscopic level and are able to simulate and predict the future behavior of the overtaking vehicle in real traffic flow. In these models, the kinematic features of Driver-Vehicle-Units (DVUs) such as distance, velocity, and acceleration are used. Unlike the previous models, where some variables of the two involved vehicles are considered to be constant, in this paper, instantaneous values of the variables are considered. The first model predicts the future value of the longitudinal acceleration and the movement angle of the overtaking vehicle. The other model predicts the overtaking trajectory for the overtaking vehicle. The second model is designed for two different vehicle classes: motorcycles and autos. Also, the result of the trajectory prediction model is compared with the result of other models. This comparison provides a better chance to analyze the performance of this model. Using the field data, the outputs of the MANFIS models are validated and compared with the real traffic dataset. The simulation results show that these two MANFIS models have a very close compatibility with the field data and reflect the situation of the traffic flow in a more realistic way. These models can be used for all types of drivers and vehicles and also in other roads and are not limited to certain types of situations. The proposed models can be employed in ITS applications and the like.

*Keywords:* Overtaking Maneuver, ANFIS, Modeling, Intelligent Automation.

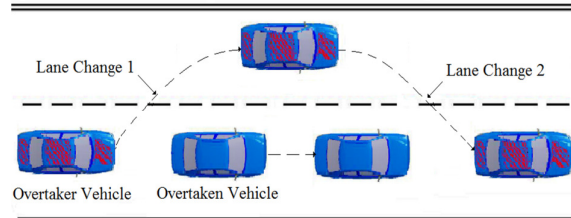
## 1. Introduction

Driver behavior is an issue that contributes directly or indirectly to the traffic congestion and safety on the road. These behaviors can be categorized into three main behaviors; car following [1], lane changing [2] and overtaking. Here, the concentration is on the overtaking behavior as a challenging behavior on highways in comparison with other driving behaviors like car following and lane

changing. In a microscopic perspective, overtaking can keep the velocity of the high-speed vehicle; in a macroscopic perspective, overtaking can improve the traffic flow rate by reducing the negative impact generated by low-speed vehicle. Overtaking is a complex driving behavior since it includes observing, information processing, decision making, planning and maneuvering. An overtaking maneuver consists of three phases: a) diverting from the original lane, b) driving straight in the adjacent lane, and c) returning to the lane [3]. The phases of the overtaking

maneuver are shown in Fig. 1 [4]. These three phases can be called in short: lane changing, overtaking and

returning. From this point, it is indicated that the relation between lane changing and overtaking is



**Fig1.** The overtaking maneuver and its two lane changes [4].

intimate, lane changing is an important part of overtaking process, and it is the base of overtaking, because it is necessary to change the lane before overtaking [5].

Using intelligent systems in modeling the behavior of vehicles is one of the approaches to make overtaking maneuvers safer. To be able to design or develop these solutions, accurate overtaking maneuver data are required. These data are useful for the development of traffic micro simulations models [6]. Models developed at microscopic levels, such as the models presented here, are being progressively used more and more by ITS specialists to develop new transportation achievements.

In this paper, two innovative multi-input multi-output ANFIS (MANFIS) models for modeling and prediction of the driver-vehicle unit (DVU) behavior in overtaking scenarios are presented. The remaining parts of this paper are organized as follows: Section II brings in a brief review on previous studies of overtaking modeling. Section III starts with an introduction on the adaptive neuro-fuzzy inference systems (ANFIS), followed by the introduction of multi-input multi-Output adaptive neuro-fuzzy inference systems (MANFIS). Next, the new MANFIS overtaking models are presented. In sections IV, these models are evaluated through different error criteria, and the conclusion is given in Section V.

## 2. Brief Review on Overtaking Models

The study on overtaking behavior has not been as extensive as the study on other driving behaviors such as car following or lane changing. This is due to the high complexity of this maneuver. The available studies concentrate on diverse aspects of this behavior. The main object of some of these studies was this behavior, but some were focused on other driving behaviors and just mentioned a little about overtaking. Some of these studies are mentioned here

in summary. Mahdi stated that the overtaking maneuver commenced when the overtaking vehicle first crossed the centerline and completes when the vehicle is clear of the opposing traffic lane [7]. Matson and Forbes used photographic techniques to measure the distances between the overtaking and the overtaken vehicle at the start and the end of the overtaking maneuver. So they were able to calculate the overtaking distance [8]. Roozenburg suggested that there are a number of input variables to develop a mathematical model of the overtaking behavior. The variables can be the impeder vehicle speed, oncoming vehicle speed, decision time of passer, headway between passer and impeder at start of the maneuver, safety margin between passer and oncoming vehicles at completion of maneuver and vehicle acceleration [9]. Mota believed that one of the reasons that the overtaking maneuver is a risky one is due to the lack of the driver's attention. The driver's focus is usually on his way forwards or sometimes he (or she) does not use the rear-view mirror [10]. Crawford has concluded that the drivers should not hesitate to commence the overtaking maneuver as it has a negative impact on road safety and they will take a longer time to act [11]. Gordon and Mart claimed that drivers are unable to estimate the overtaking distances and safety margins correctly because these calculations depend on the speed of the involved vehicles especially the overtaken vehicle [12]. Jenkins et. al. studied overtaking maneuver on a two-way two-lane roadway. They classified this behavior on the basis of a quantitative description of overtaking behavior by analyzing data collected during a overtaking experiment conducted in a driving simulator [13]. Jamson et al. investigated how mandatory and voluntary intelligent speed adaptation might affect a driver's overtaking decisions on rural roads, by presenting drivers with a variety of overtaking scenarios designed to evaluate both the frequency and safety of the maneuvers [14]. Bar-Gera et al. assessed the speed differential threshold-if there

is one-at which drivers decide to overtake a lead vehicle [15]. Hegeman et al. studied a microscopic overtaking assistant was developed to support drivers in judging whether or not an overtaking opportunity can be accepted based on the distance to the next oncoming vehicle [16]. Clarke et. al studied overtaking road accidents involving overtaking maneuvers. They distinguished ten types of overtaking accidents and discussed three in detail: collision with a right-turning vehicle, which tends to occur either because a young driver makes a faulty overtaking decision, or an older driver makes a faulty right turn; head-on collision, and the 'return-and-lose-control' accident, which is associated particularly with young drivers [17]. Farah et al. tested the hypothesis that the frequency of overtaking maneuvers on a driving simulator is associated with a faulty decision making style in the Iowa Gambling Task (IGT), a popular decision task employed for assessing cognitive impulsivity [18].

Another category of studies on overtaking present a model for different parameters of this behavior. Due to the variety of the factors that affect this maneuver, the presented models consider different factors. Besides, the approaches to study this behavior are different. Cellular automata modeling [19-24] and differential equation modeling [25, 26] are examples of the main approaches to study overtaking. In addition, the system theoretic approach and the neural network method are applied to study the human operating behavior in overtaking procedure [3, 27, 28]. Recently, the overtaking distance-based approach has also attracted attention [29-32]. In this section, a brief review on some of the previously proposed overtaking models is presented.

In 2003, Naranjo et al. offered a rule for overtaking distance based on the least square method. The inputs of the rule were the velocity of the two involved vehicles in an overtaking maneuver [33]. In 2004, Shamir offered a smooth and ergonomic optimal lane-change trajectory to be used under normal conditions for overtaking maneuvers. The relatively simple mathematical model for each lane-change trajectory was based on minimizing the total kinetic energy during the maneuver, superimposed on a "minimum-jerk trajectory" [34]. In 2005, Hassan developed a mathematical model based on the overtaking parameters which affect the behavior. Overtaking vehicle speed was chosen as a dependent variable since it describes the behavior of the overtaking drivers and it depends on the other variables. The best subset regression method was chosen to select the independent variables which entered the relationships. It concluded that there were only five factors that affected the overtaking

traffic simulation of the potential effects of an overtaking assistant for two-lane rural roads. The maneuver. These factors were the speed of the overtaken vehicle, decision time, start headway, overtaking distance and acceleration of the overtaking vehicle. Then, a mathematical model was developed to calculate the overtaking vehicle speed using these factors [35]. In 2007, Tang et al. presented three rules for the overtaking maneuver. These rules give the time required for completing an overtaking maneuver, the time which the overtaking vehicle loses during overtaking, and the overtaking distance of vehicle [36]. In 2010, Chen et al. presented a model based on the cellular automata method (CA method) for two-lane traffic flow. In this model, the effect of vehicular density and signal cycle time on the mean velocity and the mean overtaking times of the traffic flow were analyzed [37]. In 2008, Naranjo et. al offered a rule to estimate the distance of an overtaking maneuver [38]. In 2000, Polus et al. developed a model to estimate passing sight distance of the overtaking process [39].

As mentioned above, the study of overtaking models has been widespread. But neither of the presented models is able to present a model which is completely accordant to the real behavior. Being accordant means being compatible and close with real behavior. Previous works were presented based on mathematical equations, and only one such study presented a model based on real traffic data [35] which used the data of only ten drivers and it is said that due to the lack of data, the presented model may suffer from inaccuracy and validity issues. These models have mostly considered some assumptions to simplify their modeling. For example, all of them had considered the velocity of the overtaken vehicle to be constant during the whole maneuver. These assumptions make the performance of the previously presented models far from the real traffic behaviors. Due to the variety of the factors that affect this maneuver, each model considers different factors and offer distinctive rules. Besides, these rules are calculated according to various methods. So, complexity of this maneuver makes it difficult to present a model which is close to the real behavior to an acceptable extent. These models could be more reliable if they had considered most of the major factors that affect this behavior. In the meanwhile, it seems more beneficiary if they had taken into account the instantaneous value of the factors instead of a constant value [40]. Using instantaneous values of the variables definitely makes the performance of the models closer to real traffic behaviors so it is a beneficiary decision.

The basics of the adaptive neuro-fuzzy inference system network architecture applied for the

### 3. MANFIS Overtaking Behavior Model Design

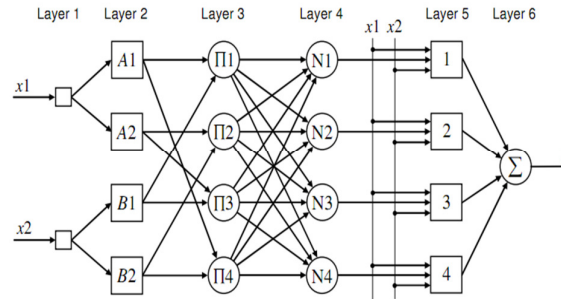


Fig2. Architecture of ANFIS model [42]

overtaking prediction system is introduced here. ANFIS enhances fuzzy inference system with self-learning capability. An adaptive network is a multilayer feed-forward network where each node performs a particular node function on incoming signals. It is characterized with a set of parameters related to that node. To reflect different adaptive capabilities, both square and circle node symbols are used. Examples of these nodes can be seen in Fig. 3. A square node (adaptive node) has parameters, while a circle node (fixed node) doesn't have. The parameter set of an adaptive network is the union of the parameter sets associated with each adaptive node. To achieve a desired input-output mapping, these parameters are updated according to given training data and a recursive least square (RLS) based learning procedure [41].

In this section, first, the MANFIS structure will be introduced. Then, details on the datasets of overtaking behavior, used to design the models, are explained. At the end, the two models improved in this study are proposed.

#### 3. A. Multiple Adaptive Neuro-Fuzzy Inference System (MANFIS)

Fuzzy logic can be a potential method dealing with structural and parametric uncertainties in the overtaking behavior. Additionally, artificial neural networks can be favorable tools providing the possibility of exploiting real observed data while developing the models. Neuro-fuzzy models, such as ANFIS, are combinations of artificial neural networks and fuzzy inference systems, simultaneously using the advantages of both methods. Integration of human

expert knowledge expressed by linguistic variables, and learning based on the data are powerful tools enabling neuro-fuzzy models to deal with uncertainties and inaccuracies [40].

The acronym ANFIS is the abbreviation for adaptive neuro-fuzzy inference system. Using a given input/output dataset, ANFIS constructs a Fuzzy Inference System (FIS) whose membership function parameters are tuned (adjusted) using either a back-propagation algorithm alone or in combination with a least square type of method. This adjustment allows the fuzzy inference systems to learn from the data they are modeling. The parameters associated with the membership functions changes through the learning process. The computation of these parameters (or their adjustment) is facilitated by a gradient vector. This gradient vector provides a measure of how well the Fuzzy inference system is modeling the input/output data for a given set of parameters. When the gradient vector is obtained, any of several optimization routines can be applied in order to adjust the parameters to reduce some error measure. This error measure is usually defined by the sum of the squared difference between actual and desired outputs [41, 42].

ANFIS model is one of the implementations of a first order Sugeno fuzzy inference system. A typical Sugeno Fuzzy rule is expressed in the following form:

$$IF x_1 \text{ is } A_1 \text{ AND } x_2 \text{ is } A_2 \dots \text{ AND } x_m \text{ is } A_m \text{ THEN } y = f(x_1, x_2, \dots, x_m). \quad (1)$$

where  $x_1, x_2, \dots, x_m$  are input variables and  $A_1, A_2, \dots, A_m$  are fuzzy sets. When  $y$  is a constant, we obtain a zero-order Sugeno fuzzy model in which the consequent of a rule is specified by a singleton, and when  $y$  is a first-order polynomial, that:

$$y = k_0 + k_1x_1 + k_2x_2 + \dots + k_mx_m \quad (2)$$

we obtain a first-order Sugeno fuzzy model. The ANFIS model is shown in Fig. 2. It is a multi-input, single-output model.

Layer 1: is the input layer. Neurons in this layer simply pass external crisp signals to Layer 2.

Layer 2: is the fuzzification layer. Neurons in this layer perform fuzzification. Fuzzification comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. The membership function is used to associate a grade to each linguistic term. Every node in this layer is a circle node labeled  $\mu$ , which multiplies the incoming signals and sends the product out. Each node output represents the firing strength of a rule.

Layer 3: is the rule layer. Each neuron in this layer corresponds to a single Sugeno-type fuzzy rule. A rule neuron receives inputs from the respective fuzzification neurons and calculates the firing strength of the rule it represents.

$$y_i = \prod_{j=1}^k x_{ji} \quad (3)$$

$$y_{\prod 1} = \mu_{A1} \times \mu_{B1} = \mu_1 \quad (4)$$

where the value of  $\mu_1$  represents the firing strength, or the truth value, of Rule 1 and according to Fig. 2,  $y_{\prod 1}$  is the output of the first node in layer 3.

Layer 4: is the normalization layer. Each neuron in this layer receives inputs from all neurons in the rule layer, and calculates the normalized firing strength of a given rule.

The normalized firing strength is the ratio of the firing strength of a given rule to the sum of firing strengths of all rules. It represents the contribution of a given rule to the final result. Thus, the output of neuron  $i$  in Layer 4 is determined as,

$$y_i = \frac{x_{ij}}{\sum_{j=1}^n x_{ij}} = \frac{\mu_i}{\sum_{j=1}^n \mu_j} = \bar{\mu}_i \quad (5)$$

Where  $y_i$  is the output of neuron  $i$  in Layer 4.

Layer 5: is the defuzzification layer. Each neuron in this layer is connected to the respective normalization neuron, and also receives initial inputs,  $x_1$  and  $x_2$ . A defuzzification neuron calculates the weighted consequent value of a given rule as:

$$y_i = x_i [k_{i0} + k_{i1}x_1 + k_{i2}x_2] = \bar{\mu}_i [k_{i0} + k_{i1}x_1 + k_{i2}x_2] \quad (6)$$

Where  $x_i$  is the input and  $y_i$  is the output of defuzzification neuron  $i$  in Layer 5, and  $k_{i0}$ ,  $k_{i1}$  and  $k_{i2}$  is a set of consequent parameters of rule  $i$ .

The architecture and learning rule of ANFIS had been described in detail by [44], and can be summarized as follows

Layer 6: is represented by a single summation neuron. This neuron calculates the sum of outputs of all defuzzification neurons and produces the overall ANFIS output  $y$  [43, 44].

$$y = \sum_{i=1}^n x_i = \sum_{i=1}^n \bar{\mu}_i [k_{i0} + k_{i1}x_1 + k_{i2}x_2] \quad (7)$$

In the ANFIS training algorithm suggested by [44], both antecedent parameters and consequent parameters are optimized. In the forward pass, the consequent parameters are adjusted while the antecedent parameters remain fixed. In the backward pass, the antecedent parameters are tuned while the consequent parameters are kept fixed. The details of the hybrid learning procedure that is used in an ANFIS are given in [41-45].

ANFIS uses the advantages of neural networks and fuzzy systems simultaneously. Some of the advantages of ANFIS in comparison with NN and fuzzy systems are: Faster convergence than typical feed forward neural networks, smaller size training set, model compactness (smaller number of rules than using labels), automatic fuzzy logic controller parametric tuning and smoothness guaranteed by interpolation. On the other hand, ANFIS has its own disadvantages too. Some of them are surface oscillations around points (caused by high partition number), spatial exponential complexity, coefficient signs not always consistent with underlying monotonic relations, not possible to represent known monotonic relations, cannot use trapezoids nor "Min", symmetric error treatment, great outliers influence, and "Awkward" interpolation between slopes of different sign.

However, the major disadvantage of ANFIS is that it can have only one output. Since multi-output systems are more frequent than single output ones, this issue influences the efficiency of ANFIS. In order to work out this problem, multi-output model can be designed by connecting several single output models [41]. In other words, putting as many ANFIS models side by side, as there are required outputs is an approach of having multiple outputs [45]. The architecture of a two-output MANFIS model is shown in Fig. 3.

MANFIS has all the advantages of ANFIS. Besides, fewer numbers of training sets are required in MANFIS to achieve the same error of single ANFIS. Therefore, faster and simpler solutions can be

obtained based on MANFIS. Also, a MANFIS model can improve to be a single-input-multi-output model [41]. In this study, MANFIS is used to effectively predict the future behavior of an overtaking maneuver [41-44].

### 3. B. Datasets

Real overtaking data from US Federal Highway Administration's NGSIM datasets are used to train the MANFIS prediction models [46]. The NGSIM datasets represent the most detailed and accurate field data collected to date for traffic micro simulation

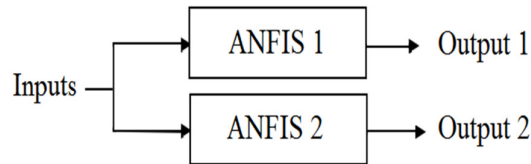


Fig3.. A two-output MANFIS structure [45]

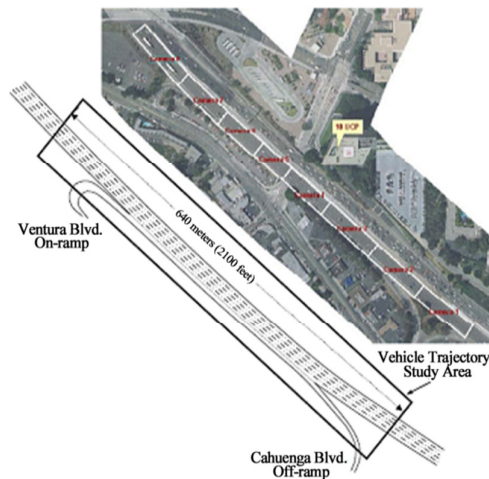


Fig4.A segment of Interstate 101 highway in Emeryville, San Francisco, California [47].

research and development. In June 2005, a dataset of trajectory data of vehicles travelling during the morning peak period on a segment of Interstate 101 highway in Emeryville (San Francisco), California has been made using eight cameras on top of the 154m tall 10 Universal City Plaza next to the Hollywood Freeway US-101. On a road section of 640m, 6101 vehicle trajectories have been recorded in three consecutive 15-minute intervals. This highway has 8 lanes. Lane 1 is the farthest left lane; lane 5 is the farthest right lane. Lane 6 is the auxiliary lane between Ventura Boulevard on-ramp and the Cahuenga Boulevard off-ramp. Lane 7 is the on-ramp at Ventura Boulevard, and Lane 8 is the off-ramp at Cahuenga Boulevard.

This dataset has been published as the US-101 Dataset. The dataset consists of detailed vehicle trajectory data on a merge section of eastbound US-101, as shown in Fig. 4. The data is collected in 0.1 second intervals. Any measured sample in this dataset has 18 features of each driver-vehicle unit in any sample time, such as longitudinal and lateral position, velocity, acceleration, time, number of lanes, vehicle class, front vehicle and etc [47].

The other dataset was published as the I-80 Dataset. Researchers for the NGSIM program collected detailed vehicle trajectory data on eastbound I-80 in the San Francisco Bay area in Emeryville, CA, as shown in Fig. 5, on April 13, 2005. The study area was approximately 500 meters (1,640 feet) in length and consisted of six freeway lanes, including a high-

occupancy vehicle (HOV) lane. I-80 highway has 9 lanes. Lane 1 is the farthest left lane; lane 6 is the farthest right lane. Lane 7 is the on-ramp at Powell Street, and Lane 9 is the shoulder on the right-side. Seven synchronized digital video cameras, mounted from the top of a 30-story building adjacent to the freeway, recorded vehicles passing through the study area. This vehicle trajectory data provided the precise location of each vehicle within the study area every one-tenth of a second, resulting in detailed lane positions and locations relative to other vehicles. A total of 45 minutes of data are available in the full dataset, segmented into three 15-minute periods.

These periods represent the buildup of congestion, or the transition between uncongested and congested conditions, and full congestion during the peak period [48].

For this study, only the lanes with straight path were used since the overtaking behavior is illegal in auxiliary lanes, on-ramps and off-ramps. In both highways, lane 1 is the farthest left lane. From the point that the first camera can see the highway, data recording starts. The NGSim program has decided to choose the farthest position of lane 1 to be the zero of coordinate X and Y.

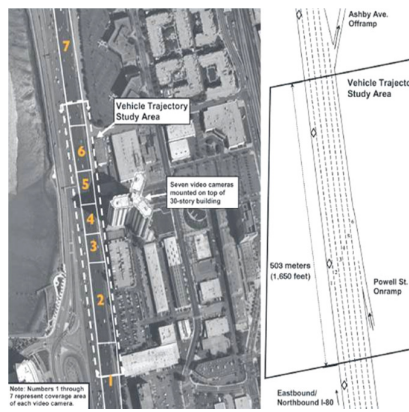


Fig5. A segment of eastbound I-80 in the San Francisco Bay area in Emeryville, California [48].

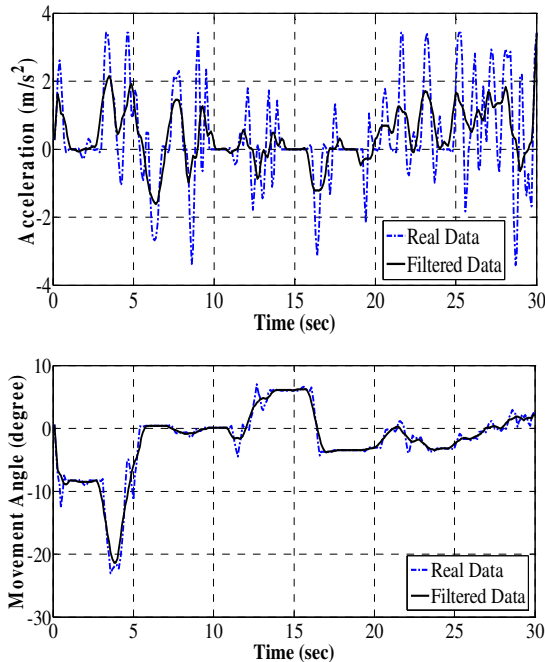


Fig6. Comparison of unfiltered and filtered data: (a) Longitudinal acceleration, (b) Movement Angle.

Finding and extracting the data of the vehicles with an overtaking behavior from the datasets was done by checking the column of the lane number of each vehicle. When a vehicle performs an overtaking behavior it travels from its initial lane to its left adjacent lane and returns back to its initial lane. So, a vehicle which had such variations in its lane number column was extracted. For this study, only the data of the vehicles in the lanes with straight path of the highways were used since the overtaking behavior is illegal in auxiliary lanes, merge points, near to a turn, curve, hill, in high-occupancy vehicle lanes, High occupancy/toll lanes, on-ramps and off-ramps according to traffic rules and regulations. Therefore, the available overtaking maneuvers were all performed in normal driving situations of the highways. But they were performed in different lanes of the highways with different velocities. So, a wide range of velocities were observed in the extracted data of overtaking vehicles which makes our models general ones.

The datasets are for two highways which have two-way traffic flows. However, consider a vehicle which is moving along in one of the lanes from one side of the highway. The opposing traffic flow does not influence the behavior of the vehicle in question, and consequently, any data corresponding to the motion of the opposing vehicles are not included in the data of this vehicle. Therefore, the passing lane is not in the forward travel lane and no opposing vehicle exists in this lane.

The data extracted from the datasets seem to be unfiltered and exhibit some noise artifacts, so these data must be filtered like [49, 51]. A moving average filter has been designed and applied to all data before any further data analysis. In the first model improved

in this study, the longitudinal acceleration and the movement angle of the overtaking vehicle is predicted. So, at first, comparison of the unfiltered and filtered data of the longitudinal acceleration and movement angle of the overtaking vehicle are shown in Fig. 6.

### 3.C. Movement Angle of the Overtaking Vehicle

The vehicle's movement angle ( $\Theta$ ), as shown in Fig. 7, is the angle between the vertical axis of the vehicle and the imaginary line through the direction of the road. This angle is different from the steering angle of the vehicle. When the overtaking vehicle deviates to the left from the straight direction of the road, the movement angle will have a negative value. But deviation to the right, leads to a positive value for this angle.

In the available datasets, there is no data available for this angle. But, it can be approximated from the coordinates of the previous and present position of the overtaking vehicle. The movement angle equation is shown in equation (8). Calculating the movement angle is not time-consuming. The lateral and longitudinal coordinates of vehicles are available in the datasets. For all the vehicles, an m-file code was written in MATLAB that computed the movement angle according to the above equation. It took less than a few seconds to compute the movement angle of all the vehicles

$$\theta = \arctan\left(\frac{x(t) - x(t-1)}{y(t) - y(t-1)}\right) \quad (8)$$

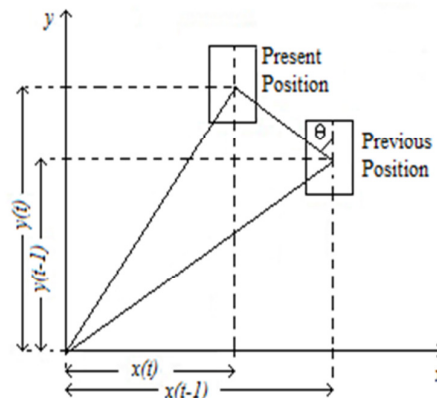


Fig7. The movement angle of the overtaking vehicle.



**Table I.** Inputs and Outputs of the Longitudinal acceleration and Movement Angle Prediction Model

Type	Parameter Name	Symbol
input	relative lateral coordinate	$\Delta x(t) = x_A(t) - x_B(t)$
input	relative longitudinal coordinate	$\Delta y(t) = y_A(t) - y_B(t)$
input	relative velocity	$\Delta v(t) = v_A(t) - v_B(t)$
input	longitudinal acceleration	$a_A(t)$
input	movement angle	$\theta_A(t)$
output	longitudinal acceleration	$a_A(t + 1)$
output	movement angle	$\theta_A(t + 1)$

**Table II.** Inputs and Outputs of the Trajectory Prediction Model

Type	Parameter Name	Symbol
input	lateral coordinate	$x_A(t)$
input	longitudinal coordinate	$y_A(t)$
input	velocity	$v_A(t)$
input	longitudinal acceleration	$a_A(t)$
input	movement angle	$\theta_A(t)$
output	lateral coordinate	$x_A(t + 1)$
output	longitudinal coordinate	$y_A(t + 1)$

### 3. D. Two MANFIS Model Design

In this study, two MANFIS model are designed. The first model predicts the longitudinal acceleration and the movement angle of the overtaking vehicle. The second model predicts the trajectory of an overtaking maneuver. As stated in section I, each of the developed models has two multiple-input-single-output ANFIS model, which make a multiple-input-multiple-output adaptive neuro-fuzzy inference system called MANFIS. To achieve an accurate prediction hybrid algorithm is used to train each ANFIS model.

From the available datasets, the data of about 1500 vehicles which had an overtaking behavior were extracted. In the development of the MANFIS prediction models, the available data are usually divided into two randomly selected subsets. The first subset is known as the training and testing dataset. This dataset is used to develop and calibrate the model. The second data subset (known as the validation dataset), which was not used in the development of the model, is utilized to validate the performance of the trained model. For this paper, 70% of the master dataset was used for training

purposes. The remaining 30% was set aside for testing the models. The designed models were tested with all the data of the test vehicles. But it was impossible to show the results of all of them in the paper. The result of the models for only one of them, chosen randomly, is shown and the error criteria for 5 of them are shown in the error table. In the following parts, each of the developed models will be described in detail [51].

#### 3. D.1 Longitudinal acceleration and Movement Angle Prediction Model

This model predicts the longitudinal acceleration and the movement angle of the overtaking vehicle. The inputs and outputs of this model are shown in TABLE I with their notations. The MANFIS system applied for this prediction model has five inputs and 2 outputs. Notice that it is assumed that the movement angle of the overtaking vehicle do not directly depend on the longitudinal acceleration. These inputs are relative lateral and longitudinal distance, relative velocity, and also the longitudinal acceleration and the movement angle of the overtaking vehicle. As mentioned before, this MANFIS model is made of

two ANFIS models that each of them predicts one of the outputs. Three gaussmf membership functions were chosen for each input and linear type was chosen for the output. Also, for the optimization algorithm, hybrid showed a better result. The rule base contains 243 fuzzy if-then rules of Takagi-Sugeno's type [52].

### 3. D.2. Trajectory Prediction Model

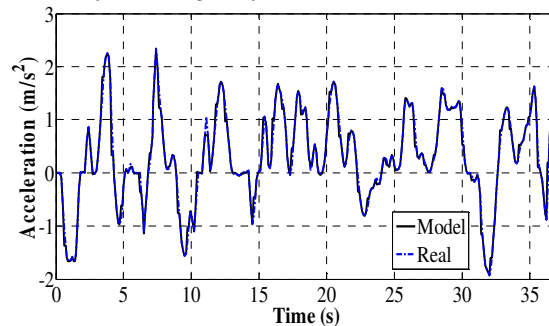
This model predicts the trajectory of the overtaking vehicle. Since the available datasets are consisted of the data of motorcycles and autos, in this part two trajectory prediction model are designed. The reason that the previous model is not designed for each of these vehicle types is due to the dynamic of a motorcycle and an auto. Studying the data of these vehicles represents that the range of the longitudinal acceleration and the movement angle of an overtaking motorcycle doesn't have much difference with the range of these parameters of an overtaking auto. Therefore, designing only one model is satisfactory for both vehicle classes. But, since the size of a motorcycle is different from the size of an auto, the overtaking trajectories will be different consequently. This difference is prominent in the lateral coordinate of the overtaking trajectory. A motorcycle can complete an overtaking maneuver by traveling only

about 0.5m to the left, but, an auto needs to travel about 3-4m to the left.

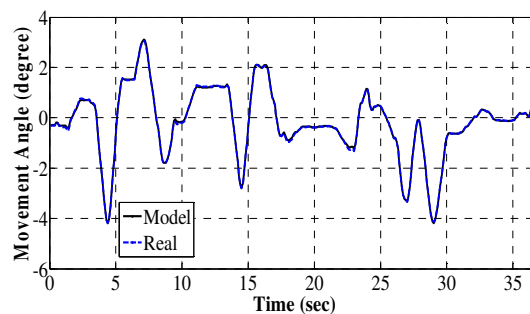
However, the structure of the ANFIS models used to make the trajectory prediction model for these two vehicle classes are the same. The only difference is in the data used to train the ANFIS models. The inputs and outputs of the trajectory prediction model are shown in TABLE II with their notations. The fuzzy inference system applied for prediction model has five inputs and 2 outputs. These inputs are lateral and longitudinal distance, velocity, longitudinal acceleration and the movement angle of the overtaking vehicle. This MANFIS model is also made of two ANFIS models that each of them predicts one of the outputs. There are three gaussmf membership functions for each input. The rule base contains 243 fuzzy if-then rules of Takagi-Sugeno's type.

## 4. Discussion and Results

To evaluate the competence of MANFIS estimator systems, the validation dataset is used for each developed model. The matrix of the validation data is divided to two groups, the input columns and the output columns. The input columns are fed as the inputs of the models. Then, the output of the model is compared to the real output, which are the output columns of the validation data. The evaluation of the proposed models is brought in the following parts.



(a)



(b)

Fig8. Comparison of the MANFIS model and real data: (a) longitudinal acceleration, (b) Movement Angle.

#### 4.A. Evaluation of the Longitudinal Acceleration and Movement Angle Prediction Model

The comparison of the output of the longitudinal acceleration and movement angle prediction model with real data is shown in Fig. 8. This figure shows the data of only one test vehicle and this data wasn't in the training dataset of the model.

To examine the performance of the developed model, various criteria are used to calculate errors. The mean square error (MSE) of an estimator, according to equation (9), is one of the many ways to quantify the difference between values implied by an estimator and the true values of the quantity being estimated. Root mean squares error (RMSE), according to equation (10), is a criterion for comparing error dimension in various models. The normalized mean square error (NMSE), according to equation (11), is an estimator of the overall deviations between predicted and measured values. The mean absolute error (MAE), according to equation (12), is a quantity used to measure how close forecasts or predictions are to the eventual outcomes. The Symmetric mean absolute percentage error (SMAPE), according to equation (13), is an accuracy measure based on percentage (or relative) errors. In these equations,  $x_i$  shows the real value of the variable being modeled (observed data),  $\hat{x}_i$  shows the real value of variable modeled by the model and  $\bar{x}$  is the real mean value of the variable and N is the number of test observations [53].

$$MSE = \frac{1}{N} \sum_{i=1}^N (x_i - \hat{x}_i)^2 \quad (9)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \hat{x}_i)^2} \quad (10)$$

$$NMSE = \frac{\sum_{i=1}^N (x_i - \hat{x}_i)^2}{\sum_{i=1}^N (x_i - \bar{x})^2} \quad (11)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |\hat{x}_i - x_i| \quad (12)$$

$$SMAPE = \frac{1}{N} \sum_{i=1}^N \frac{|x_i - \hat{x}_i|}{|x_i + \hat{x}_i|} \quad (13)$$

Errors in modeling the longitudinal acceleration output and the movement angle output considering these criteria are summarized in TABLE III and TABLE IV. The last column of tables shows the mean value of each error criteria.

#### 4. B. Evaluation of the Trajectory Prediction Model

The comparisons of the output of the MANFIS model with real data and are shown below. Fig. 9 shows the trajectory of the two vehicle classes for one vehicle of each class. This figure shows the data of only one test motorcycle and one test auto and these data weren't in the training dataset of the models. We could simply show the result of the trajectory model for the whole test dataset. But since the overtaking behavior is a behavior with a specific trajectory, we wanted to show the exact trajectory of each vehicle to present the longitudinal and lateral displacement more clearly. In other words, showing the trajectory of each vehicle separately shows the 3 phases of the behavior better.

**Table III.** Result of Error for MANFIS Overtaking Model: Longitudinal Acceleration

Criteria	Test 1	Test 2	Test 3	Test 4	Test 5	Mean
MSE	0.0034	0.0580	0.0447	0.0351	0.0300	0.034
RMSE	0.0581	0.2409	0.2114	0.1873	0.1731	0.174
NMSE	0.0020	0.1663	0.0387	0.0372	0.0216	0.053
MAE	0.0303	0.2457	0.0610	0.1455	0.1270	0.122
SMAPE	0.1170	0.0977	0.0515	0.2464	0.3258	0.168

**TABLE IV .**Result of Error for MANFIS Overtaking Model: Movement Angle

Criteria	Test 1	Test 2	Test 3	Test 4	Test 5	Mean
MSE	6.7e-005	0.0405	0.0028	0.0095	0.0115	0.013
RMSE	0.0082	0.2012	0.0530	0.0973	0.1072	0.093
NMSE	4.8e-004	0.0456	0.0062	0.0216	0.0144	0.018
MAE	0.0061	0.1003	0.0221	0.0677	0.0823	0.056
SMAPE	0.0110	0.1002	0.0912	0.1005	0.5661	0.174

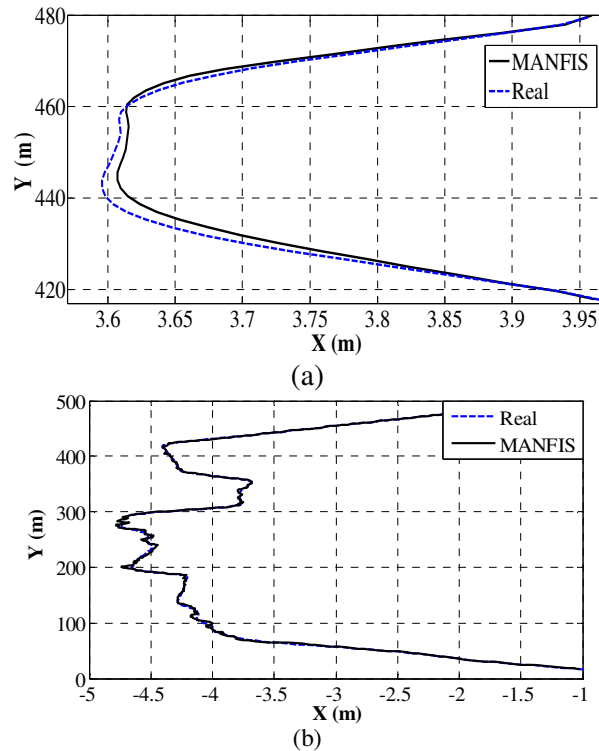


Fig9. Trajectory of an overtaking behavior for only one vehicle of the validation dataset: (a) motorcycle, (b) auto.

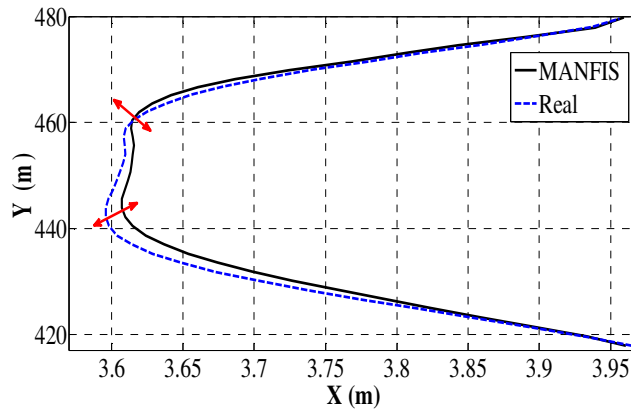


Fig10. Distinguishing the 3 phases of an overtaking trajectory

The 3 phases of an overtaking behavior can be distinguished from Fig. 10. In Fig. 10(a), the start point of the figure with the coordinates of  $x=3.93$  and  $y=419$ , is the start point of phase one. As the motorcycle continues to go to the left lane (mostly changing the  $x$  coordinate), it is the 1st phase of the behavior, up to point around  $x=3.62$  and  $y=445$ . The second phase starts as the motorcycle starts to change its  $y$  coordinate from this point to around point

$x=3.43$  and  $y=460$ , it is moving straight and it can be said that it is moving in a straight path and only its  $y$  coordinate is changing. From this point to the end of the trajectory ( $x=3.92$  and  $y=480$ ), the motorcycle is going back to its initial lane, and it can be said that mostly, only its  $x$  coordinate is changing in comparison with its  $y$  coordinate. These phases are separated from each other in Fig. 10 by double arrows.

The optimal trajectory model offers three different trajectories for the three different phases of an overtaking maneuver. The lane change for the third phase of the maneuver has the equations which are mentioned in equation (2) and (3). For the trajectory of the first lane change, Shamir only used symmetry and time reversal. And for the trajectory of the second phase, which is the passing phase, Shamir defines the optimal time and distance which must be traveled. These parameters are calculated according to equations (6) and (7). Then he assumed that in this phase the lateral displacement is the width of the lane ( $W$ ), and the longitudinal distance is equal to the optimal distance ( $D$  (b)).

Notice that the MANFIS model and the optimal model present the lateral and longitudinal coordinates with unlike parameters. In the MANFIS model, lateral coordinate is shown by  $x$  and the longitudinal coordinate is shown by  $y$ . But in the optimal model, the names of the parameters are vice versa. Another point that must be noted is that in the optimal model, the horizontal axis shows the longitudinal coordinate of the trajectory, and the vertical axis shows the lateral coordinate of the trajectory. For the data of the same auto used in Fig. 10 (b), the optimal trajectory model offers the trajectory shown in Fig. 11.

In order to compare the performance of the two models numerically, some parameters of the

trajectories shown in Fig. 10 b) and Fig. 11 are compared with the real overtaking trajectory in TABLE V.

Despite the similarity in the range of some of the parameters shown in TABLE V, major differences exist between the real and optimal trajectory. One disadvantage of the optimal model is that the lateral distance traveled is always equal to the width of the road ( $W$ ). But in reality, it does not happen as ideal as the optimal model shows. Due to this characteristic, the second phase always starts from a point with lateral coordinate equal to  $W$ . Therefore, the trajectory of the first phase always starts from a point with negative coordinate  $x$ . All the trajectories resulted by this model have this property. Because of this property, the start and final points of the trajectory are not even close to reality. Having three distinct trajectories instead of a continuous one, similar to the real trajectory, is another disadvantage of the optimal model. In addition, the optimal model isn't able to predict the trajectory for test vehicles with negative or zero longitudinal acceleration, but the MANFIS model is completely capable of predicting the trajectory for different values of the longitudinal acceleration. Also, for cases with positive longitudinal acceleration, the model does not have a proper result when the value of the longitudinal acceleration increases

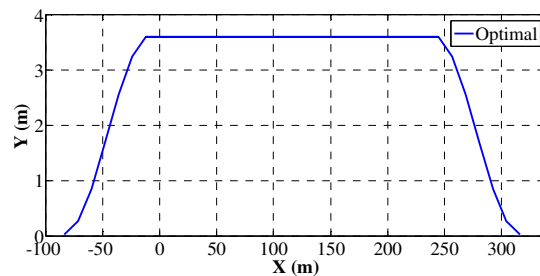
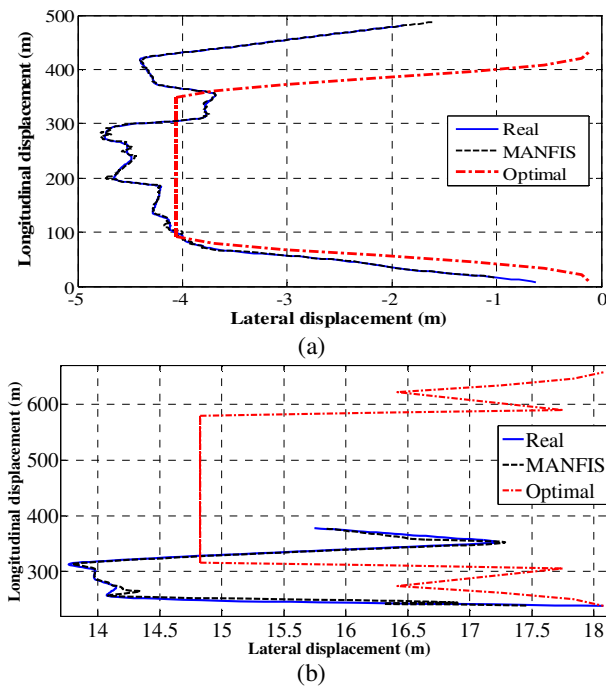


Fig11. The optimal trajectory for the same auto.

TABLE V Comparison of the parameters of the two models with real data.

arameters of the trajectories of test vehicle 1	Real data	MANFIS model	Optimal model
Time (s)	46.8	46.8	34.65
Lateral distance (m)	4.6	4.7	3.6
Longitudinal distance (m)	473	474	399
Coordinate x of the start point (m)	20.38	20.38	-0.02
Coordinate y of the start point (m)	30.29	30.31	-83.59
Coordinate x of the end point (m)	19.24	22.32	-0.02
Coordinate y of the end point (m)	516	510	316



**Fig12.** Comparison of the results of the two models with real data for one test vehicles

In these situations, the trajectory for the lane change phases of the maneuver will not be a smooth trajectory anymore. Another problem is that in the optimal model, the total time of the maneuver is not equal to the time spent in reality. One more disadvantage is about the total distance traveled during the maneuver. The optimal model isn't able to predict the total distance correctly. So, in some cases the distance is more than the real distance, and sometimes it is less.

Here, in order to have a better comparison between the trajectories of the two models, the trajectory of the optimal model is rotated. Then, the trajectory is shifted to the start point of the real trajectory. After rotation, in both trajectories, the horizontal axis shows the lateral displacement, and the vertical axis shows the longitudinal displacement. The comparison of the output of the two models with real data for the same test vehicle used in Fig 10 (b) and Fig. 11, is shown in Fig. 12 (a). In Fig 12 (b), another comparison of the three trajectories is shown. For this case, the longitudinal acceleration of the test vehicle was more than the previous case. As it is shown, the trajectory of the lane change phases is not a uniform trajectory.

To examine the performance of the developed models, various error criteria are used. But since the total time of the trajectory of the optimal model is not equal to the time in real trajectory data, it is not

possible to calculate these criteria for this model. So, the results of these criteria are only calculated for the MANFIS model.

To evaluate the performance of the trajectory prediction MANFIS model, two-variable error criteria must be used. It is emphasized here that this model has two outputs, namely, the lateral and longitudinal coordinates. To evaluate the true accuracy of the trajectory, hence, it is necessary that one uses a two-variable evaluating criteria. If simply a one-variable error criterion such as the MSE or MAPE for each of the outputs is used for the trajectory evaluation, the result for the trajectory can be misleading. Therefore, it is not logical to evaluate the accuracy of the lateral and longitudinal coordinates separately for a total trajectory evaluation. An acceptable trajectory is one which matches the real trajectory considering both lateral and longitudinal coordinates. So, two-variable error criteria will be used.

The average absolute horizontal transport deviation (AHTD), according to equation (14), shows the mean deviation between a modeled trajectory and the corresponding true trajectory. The trajectory based on field data is considered as true trajectory. Another useful statistical concept is the average relative horizontal deviation (RHTD), according to equation (15). This is defined as the ratio between the absolute transport deviation and the mean total travel distance of the true trajectory ( $L_H(t)$ ), according to equation

(16). In these equations,  $X_n(t)$  and  $x_n(t)$ , respectively, show the real and model value of the coordinate x. In addition,  $Y_n(t)$  and  $y_n(t)$  show the real and model value of the coordinate y. N is the number of test observations at travel time t [54, 55].

$$AHTD(t) = \frac{1}{N} \sum_{n=1}^N \{ [X_n(t) - x_n(t)]^2 + [Y_n(t) - y_n(t)]^2 \}^{1/2} \quad (14)$$

$$RHTD(t) = \frac{AHTD}{L_H(t)} \times 100 \quad (15)$$

$$L_H(t) = \frac{1}{2} \sum_{i=1}^t \{ [X_n(t_i) - X_n(t_{i-1})]^2 + [Y_n(t_i) - Y_n(t_{i-1})]^2 \}^{1/2} + \{ [x_n(t_i) - x_n(t_{i-1})]^2 + [y_n(t_i) - y_n(t_{i-1})]^2 \}^{1/2} \quad (16)$$

Errors in modeling the trajectory of the overtake maneuver of a motorcycle and an auto, considering these criteria are summarized in TABLE VI and TABLE VII. The last row of tables shows the mean value of each error criteria.

In order to complete validating the designed models, it is necessary to state several points about the designed models. First of all, it is stressed that the datasets used in this study for the training of the Neural Network are quite comprehensive, rather global, and inclusive of a good many number of driving situations, initiated and provided by the well-respected, well-known entity known as the US Department of Transportation (US DOT) Federal Highway Administration (FHWA) through a program known as The Next Generation Simulation (NGSIM) program. These datasets record the behavior of a Driver-Vehicle-Unit (DVU) without the awareness of drivers. So, each driver acted as usual as he/she did, therefore the dataset was comprised from different patterns of drivers. As a result, the range of the variables was as widespread as the number of DVUs in the recording area. These datasets consist of

detailed vehicle trajectory, wide-area detector, and supporting data for researching driver behavior. The vehicle trajectory data, which were collected using digital video cameras, are particularly valuable due to the unprecedented level of detail and accuracy. For example, the precise location of each vehicle on a 0.5- to 1.0-kilometer section of roadway is recorded every one-tenth of a second. As a result, transportation practitioners will be able to develop Driver-Vehicle Unit (DVU) behavior models using high-quality, real-world datasets. Although the datasets were recorded during the peak field, they have recorded a wide range of driver behaviors which approximately contain most of the possible behaviors in a highway. In other words, different ranges of important variables like acceleration, velocity and position have been recorded. As a result, the developed models in this study have satisfactory responses for mostly all the test vehicles that were fed to the models. As such, it is the strong opinion of the authors that the MANFIS models devised here in this work will be able to respond properly in most, if not all, driving cases. Therefore, the models presented here may not be required to be retrained with new datasets and can be used in an “archival” manner, as put forth by the respected reviewer. 2- The very concern of the need to retrain a Neural Network for “new” situations in an offline training session is a well-known drawback in the general employment of Neural Network concept. This is a drawback of Neural Networks in general, and not of the present paper. In fact, we state that, though not used in our current work, we present a possible solution to this NN problem or concern by proposing that one can use an ONLINE neural network training methodology (such as a recurrent or dynamic network) to train the NN while it is being used and as the data is being generated in real time.

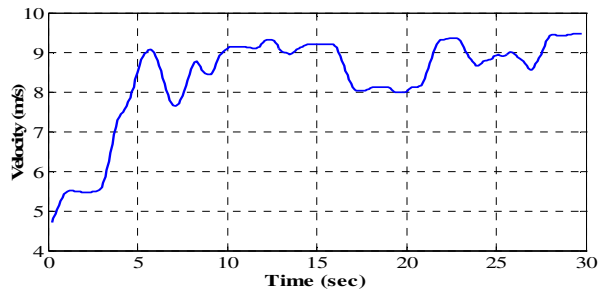
Also, it is also necessary to explain that all the figures in this paper show only the data of one single overtaking behavior and they are not the data of the entire test dataset

TABLE VI. Error for the MANFIS Overtaking Model for Motorcycle

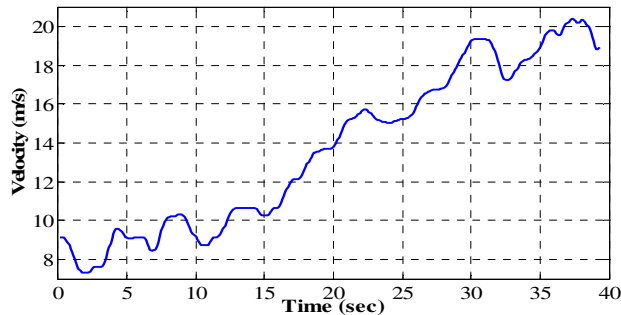
Error Criteria	AHTD (m)	RHTD (%)
Test motorcycle 1	0.0967	0.2336
Test motorcycle 2	0.0073	0.0118
Test motorcycle 3	0.1225	0.4589
Test motorcycle 4	0.0402	0.0855
Mean	0.0665	0.1974

**TABLE VII.** Error for the MANFIS Overtaking Model for Auto

Error Criteria	AHTD (m)	RHTD (%)
Test Vehicle1	0.0049	0.0034
Test Vehicle2	0.0065	0.0105
Test Vehicle3	0.0019	3.5e-004
Test Vehicle4	0.0139	0.0028
Mean	0.0068	0.0042



(a)



(b)

**Fig13.** Comparison of the range of velocities, (a) test vehicle of Fig. 7, (b) test vehicle of Fig. 9.

About the range of variations of the movement angle, it should be stated that this range is different for each vehicle. It is due to the different ranges of velocity that each vehicle has. With lower velocities, drivers need to increase the movement angle to change lane. But with higher velocity, drivers can change lane with a slight difference in the range of movement angle. For example, for the test vehicle used for Fig. 7 which had a movement angle between 10 to -30 degrees, the velocity of the vehicle is

between 4.5m/s to 9.5m/s (16.2-34.2km/h). But for the test vehicle of Fig. 9, the velocity of the vehicle is between 7m/s to 21 m/s (25.2-72km/h). In Fig. 13, the velocities of these two vehicles are shown.

### Conclusion

In this paper, two novel overtaking models were proposed. These models consider important factors such as longitudinal and lateral distance, velocity, and



the longitudinal acceleration and movement angle of the overtaking vehicle. Satisfactory performance of the proposed model was demonstrated through comparisons with real traffic data. The simulation tests for different experimental configurations have shown the effectiveness of the proposed overtaking models. This effectiveness was in prediction of the future value of the longitudinal acceleration and movement angle of the overtaking vehicle. Using the instantaneous value of the parameters to predict the future value of them is the prominent aspect of the proposed overtaking model.

By using the results of the suggested model driver assistant systems equipped with appropriate sensors, can estimate the longitudinal acceleration and movement angle for its future movement.

The proposed method can be recruited in driver assistant devices, safe distance keeping observers, collision prevention systems and other ITS applications.

Satisfactory performance of the proposed model was demonstrated through comparisons with real traffic data and the result of another model named the optimal model.

As table II shows, the MANFIS overtaking model based on instantaneous value of the parameters is very close to the real trajectory data. This result shows that these MANFIS models have a strong capability with respect to other models.

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