2012 IEEE International Conference on Vehicular Electronics and Safety
July 24-27, 2012. Istanbul, Turkey

The Effects of the Cornering Stiffness Variation on Articulated Heavy Vehicle Stability*

S. Hamed Tabatabaei, Ahmad Zahedi, and Alireza Khodayari, *Member, IEEE*

Abstract—**In this paper, the effects of the Cornering Stiffness (CS) variation on directional stability of the Articulated Heavy Vehicle (AHV) are examined through analysis and simulations. The CS is one of the most important parameter which has a considerable effect on vehicle dynamic behavior. On the other hand, due to the physical nature of the tyre, the CS varies significantly in response to the braking or driving torques applied to the wheels. Therefore, there is a vital need to study the effect of the CS variation on directional stability of AHVs. A linear planar model of articulated vehicle is used to investigate the effects of CS variation on directional stability. Furthermore, the results derived by this analysis are confirmed through lane change maneuver simulation by a full nonlinear planar model of the articulated vehicle. The results achieved in this paper can be used to develop an appropriate procedure for braking or driving forces distribution.**

I. Introduction

Heavy vehicles, due to their large weight and dimensions, are more likely to unstable motion than most other vehicles. An unstable motion of the articulated heavy vehicle is a critical issue that may lead to fatal accidents. A review of heavy vehicle accident in the U.S and Canada revealed that heavy trucks were involved in 28% of single vehicle accidents, as compared with 19% for passenger vehicles [1]. In contrast to light vehicles, less attention has been paid to stability issues of heavy vehicles, particularly to the articulated heavy vehicles (AHVs). An AHV consists of a truck unit and one or multiple towed unit(s), called trailer(s). Due to several economic issues, the role of AHVs in transportation of freight is growing rapidly. AHVs move more payloads with lower tare weight than single unit vehicles [2]. Furthermore, the reduction of drivers will reduce the operating cost substantially.

It has been established that directional stability limits of AHVs are influenced by the varying of vehicle physical parameters such as weight, wheel base and center of gravity height etc. In 1965, Jindra showed that the yaw oscillation of the trailer unit increase with an increase the yaw moment of inertia of the trailer body [3]. Ervin et al. [4] investigated the impact of size and weight variables on the stability and control of heavy trucks and tractor- trailer combinations. The influence of multiple axles and articulation points of AHVs on the directional dynamics have been examined by Fancher in 1989[5]. He derived a handling equation to analyze the steady state turning characteristic of vehicle with multiple axels.

An algebraic equation for the stability boundary as a function of the vehicle parameters has been proposed by Eliss [6]. Kaneko and Kageyama studied the stability of a tractor-semitrailer combination during braking[7]. The effect of the geometric location of the fifth wheel on AHV stability has been investigated by Dahlberg and Wideberg in 2004 [8]. They showed that as the fifth wheel is moved rearward, the vehicle response become highly non-linear and unpredictable. Hac et al. [9] through stability analysis and simulation model examined the advantages and limitations of two active brake control methods in Articulated Vehicles. More recently, Feng et al.[10], used the root locus analysis to determine the degree of influence of some physical parameters on AHV stability.

One important factor for the stability analysis that is not so well covered in the literature is the tyre Cornering Stiffness. There is a lack of technical researches investigating precisely the influence of cornering stiffness variation on directional stability of AHVs. However the variation of the cornering stiffness certainly affects the vehicle dynamic behavior. On the other hand, due to the physical nature of the tyre, the cornering stiffness varies significantly in response to the braking or driving torques applied to the wheels. Based on the foregoing studies, there is a vital need to study the effect of the cornering stiffness variation on directional stability of AHVs. Therefore, this paper presents a systematic method to fulfill this task.

The rest of this paper is organized as follows: a linear and nonlinear planar model of the articulated heavy vehicles is developed in section 2. Next, the braking/driving torque effect on CS variation is studied in section 3. In section 4, a linear planar model of articulated vehicle is used to investigate the effects of CS variation on directional stability. Furthermore, the results derived from the previous analysis are confirmed through lane change maneuver simulated by full nonlinear planar model of the articulated vehicle. Finally, the results of this paper are presented in section 5.

*Resrach supported by South Tehran branch of Islamic Azad University.
S. Hamed Tabatabaei is with the Department of Mechanical Engineering, South Tehran Branch, Islamic Azad University, Ahang Ave., Tehran, Iran (corresponding author to provide phone: 98-021-44212315; e-mail: hamedtaba@ yahoo.com).
Ahmad Zahedi is with the Department of Mechanical Engineering, Firouzkooh Branch, Islamic Azad University, Firouzkooh, Iran
Alireza Khodayari is with the Department of Mechanical Engineering, K. N. Toosi University of Technology.

II. Vehicle Model

In this section two models, nonlinear planar model and linear planar model have been developed. The nonlinear model is used to test the predicted results derived from the stability analysis and the linear model is used to study the stability of the vehicle.

A. Nonlinear vehicle model

The fourteen degrees of freedom model reflecting the directional characteristics of the articulated vehicle is used to test the proposed approach. The articulated vehicle was modeled using two rigid bodies including the tractor with front steerable axle supplying by the driver and the semitrailer with two axles. As shown in Fig.1, the degrees of freedom are longitudinal, lateral and yawing motion of tractor, the articulation angle between the tractor and the semitrailer and the rotational motion of each wheel.

Three coordinate systems for the model are considered. The first is the inertial coordinate system (X_nY_n) fixed to the ground, the second is tractor coordinate system (X_tY_t) fixed to the tractor's centre of gravity and the third is semitrailer coordinate system (X_sY_s) mounted to the semitrailer CG. The X_tY_t and X_sY_s coordinates moving with the tractor and the semitrailer units have yaw velocities of r_t and r_s, respectively. So the rate of the articulation angle can be represented by:

$$\dot{\gamma} = r_s - r_t \tag{1}$$

The motion equations representing longitudinal, lateral and yaw dynamics of the tractor and semitrailer units can be simply derived. Eliminating the coupling forces from these equations leads to the following four relations represent the planar motions of the articulated vehicle in X_tY_t coordinate:

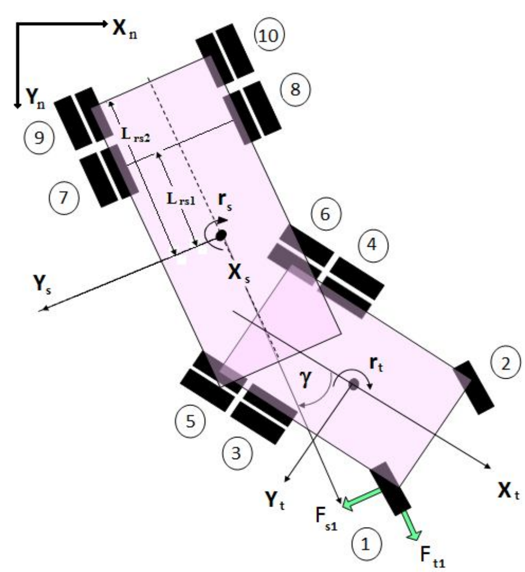

Figure 1. Fourteen degrees of freedom articulated vehicle model

$$\begin{aligned}&(m_t + m_s)(\dot{V}_{xt} - r_t V_{yt}) + \\ &m_s\left[L_{wt} r_t^2 + L_{fs} r_s^2 \cos\gamma + L_{fs}\dot{r}_s \sin\gamma\right] = F_{xtt} + F_{xts}\end{aligned} \tag{2}$$

$$\begin{aligned}&(m_t + m_s)(\dot{V}_{yt} + r_t V_{xt}) + \\ &m_s\left[L_{fs} r_s^2 \sin\gamma - L_{wt}\dot{r}_t - L_{fs}\dot{r}_s \cos\gamma\right] = F_{ytt} + F_{yts}\end{aligned} \tag{3}$$

$$I_{zt}\dot{r}_t + m_t(\dot{V}_{yt} + r_t V_{xt})L_{wt} = M_{zt} + F_{ytt}L_{wt} \tag{4}$$

$$\begin{aligned}&I_{zs}\dot{r}_s - m_t(\dot{V}_{xt} - r_t V_{yt})L_{fs}\sin\gamma + m_t(\dot{V}_{yt} + r_t V_{xt})L_{fs}\cos\gamma \\ &= M_{zs} - F_{xtt}\sin\gamma L_{fs} + F_{ytt}\cos\gamma L_{fs}\end{aligned} \tag{5}$$

Where F_{xtt}, F_{ytt} and M_{zt} are the total longitudinal force, lateral force and yaw moment acting on the tractor unit in tractor coordinated system (X_tY_t); F_{xts}, F_{yts} and M_{zs} are the total longitudinal force, lateral force and yaw moment acting on the semitrailer unit in tractor coordinated system (X_tY_t). The wheel rotational dynamics can be expressed as:

$$I_w\dot{\omega}_i = -R_w F_{ti} + T_i \tag{6}$$

Where T_i is the applied torque to the wheel, F_{ti} is the tractive force, ω_i is the angular velocity of each wheel. The tractive and side forces acting on the articulated vehicle are generated at the contact path between tyre and road. In this paper, the Dugoff model is used to simulate these forces [11]. The inputs of the model are the side slip angle, wheel slip ratio and normal tyre load. Finally, a SIMULINK program has been developed to simulate the motion of the articulated vehicle.

B. Linear vehicle model

The linear model used to study the directional stability, is a single track 4-th order model shown in Figure.2. As schematically shown, it is assumed that one tyre is located at the centre of the tractor front axle, tractor rear axle and semitrailer rear axle. Furthermore, the front wheel steering angle, the articulation angle, and the tyre slip angles are assumed to be small. In addition, the tyre lateral forces as linear functions of the tyre slip angles are assumed.

Under the above assumptions, the linear motion equations of the articulated vehicle can be represented as:

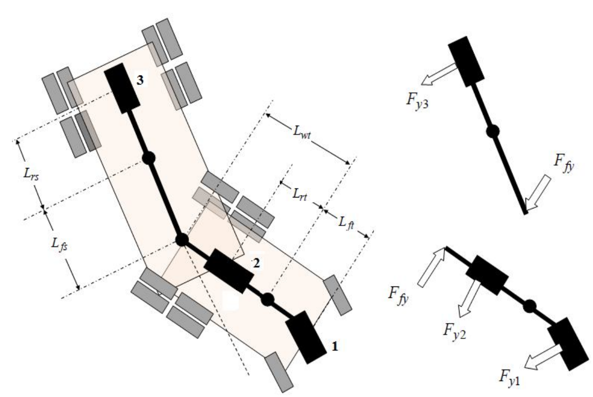

Figure 2. Linear model of the articulated vehicle

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} V_{yt} \\ r_t \\ \dot{\gamma} \\ \gamma \end{Bmatrix} + \begin{Bmatrix} C_1 \\ C_1(L_{ft} + L_{wt}) \\ 0 \\ 0 \end{Bmatrix} \delta_f = \begin{bmatrix} m_s + m_t & -m_s(L_{wt} + L_{fs}) & -m_s L_{fs} & 0 \\ m_t L_{wt} & I_{zt} & 0 & 0 \\ -m_s L_{fs} & I_{zs} + m_s L_{fs}(L_{fs} + L_{wt}) & m_s L_{fs}^2 + I_{zs} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{Bmatrix} \dot{V}_{yt} \\ \dot{r}_t \\ \ddot{\gamma} \\ \dot{\gamma} \end{Bmatrix} \quad (7)$$

Where

$$d_5 = L_{wt} + L_{fs} + L_{rs} \quad (8)$$
$$d_6 = L_{fs} + L_{rs}$$
$$a_{11} = -\frac{c_1 + c_2 + c_3}{V_{xt}}$$
$$a_{12} = \frac{c_2 L_{rt} - c_1 L_{ft} + c_3 d_5 - (m_t + m_s)V_x^2}{V_x}$$
$$a_{13} = \frac{c_3 d_6}{V_{xt}}$$
$$a_{14} = c_3$$
$$a_{21} = -\frac{c_1(L_{ft} + L_{wt}) + c_2(L_{wt} - L_{rt})}{V_{xt}}$$
$$a_{22} = \frac{-c_1(L_{ft}^2 + L_{ft}L_{wt}) + c_2(L_{wt} - L_{rt})L_{rt} - m_t L_{wt} V_{xt}^2}{V_x}$$
$$a_{31} = \frac{c_3 L_5}{V_{xt}}$$

Equation (7) can be expressed in a matrix form as follows:

$$A_1 x + C\delta_f = M\dot{x} \quad (9)$$

Where the state vector, x, is defined as:

$$x = [V_{yt}, r_t, \dot{\gamma}, \gamma]^T \quad (10)$$

The equation (9) can be represent in state space form by premultiplying on both side by M^{-1}:

$$\dot{x} = Ax + B\delta \quad (11)$$

where

$$A = M^{-1}A\ , \quad B = M^{-1}C \quad (12)$$

III. Cornering Stiffness Dependency with Wheel Slip Ratio

To provide a measure for comparing the cornering behavior of different tires, a parameter called cornering stiffness is used. It is defined as the derivative of the cornering force with respect to tire slip angle evaluated at zero slip angle[12].

This value can be affected by some factors which include, but are not limited to, normal load, friction coefficient, tire pressure, and wheel slip ratio. When a driving (braking) torque is applied to the tyre, the distance that the tyre travels will be less (more) than that in free rolling. This phenomenon is usually referred to as wheel slip ratio defined as follows:

$$\lambda = \frac{\omega R_e - V_t}{\max(\omega R_e, V_t)} \quad (13)$$

This ratio can vary from 0 (perfect match between wheel and vehicle speeds) to 1 regardless of the sign of the slip ratio. As shown in figure. 3, through the application of the braking or driving torque and the increasing the magnitude of the wheel slip ratio, the CS is significantly decreased.

It is important to note that this dramatic change in CS may reduce the directional stability and vice versa. In the next section, the effects of the CS variation of each axle are studied independently.

IV. Directional Stability Analysis

In this section an eigenvalue analysis is performed to study the directional stability of the articulated vehicle. To determine the system eigenvalues, the characteristic equation should be written. The steering angle in equation (11) set as zero and the system characteristic equation is defined as follows:

$$\det(sI - A) = 0 \quad (14)$$

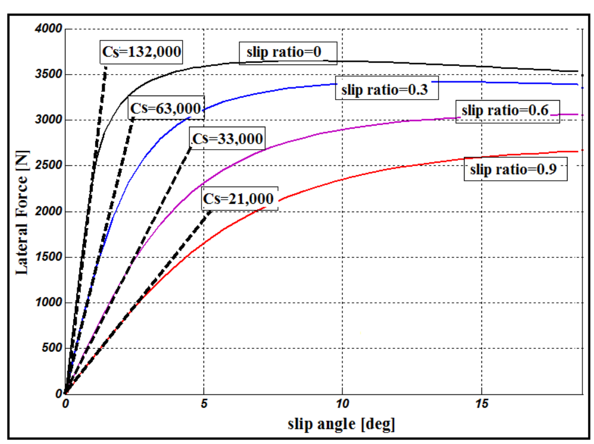

Figure 3. Cornering stiffness dependency with wheel slip ratio

Where s is the laplace operator and I is the identity matrix. It is clear that the characteristic equation is a 4th order equation. The most convenient way to analyze the effect of CS variation on vehicle stability is to plot the root locus. So, in the following subsections, the trajectories of the characteristic equation's roots are drawn as a function of the cornering stiffness. This task is performed for each axle independently to determine the influence of e CS of each axle on directional stability. It is also shown that this analysis can offer an appropriate procedure for distribution of the braking or driving torque. In the remainder of this section the cornering stiffness of the tractor front axle, tractor rear axle, and trailer rear axle are denoted as C_1, C_2, and C_3, respectively.

A. The effects of the cornering stiffness of the tractor front axle

In figure(4), the root locus is plotted as a function of the C_1. This value is reduced from 132000[N/deg] to 21000 [N/deg] based on the physical fact depicted in figure(3). According to figure (4), with significant decrease of C_1, the amount of the real part of the dominant eigenvalue is slightly reduced. Furthermore, non-significant decrease in damping ratio is observed. These observations imply that the directional stability of the articulated vehicle is not significantly affected by varying the cornering stiffness of the front axle of the tractor unit.

In order to evaluate the mentioned prediction in real applications, the following maneuver is simulated by the nonlinear model presented in section II. In this simulation scenario, braking during high speed lane change maneuver is performed. The articulated vehicle initially travels with a velocity of 100 km/h on a dry road and the steering angle made one period of sine wave with the amplitude of 3deg and frequency of 0.5 Hz. In order to decrease the CS of front axle, at the same time, a heavily braking torque applied to the front wheels. The vehicle path is shown in figure 5. It can be seen that the system still exhibits the stable response. However, additional efforts are required to return the AHV into the path. So, both analysis and simulation revealed stable response across a wide variation of CS.

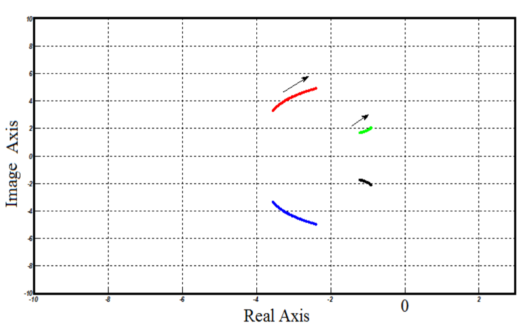

Figure 4. The effect of C_1 variation on directional stability

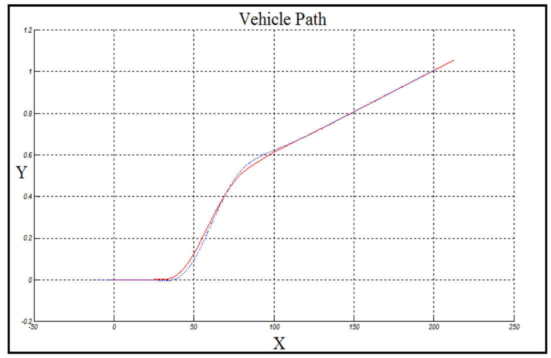

Figure 5. The vehicle path in lane change manuver

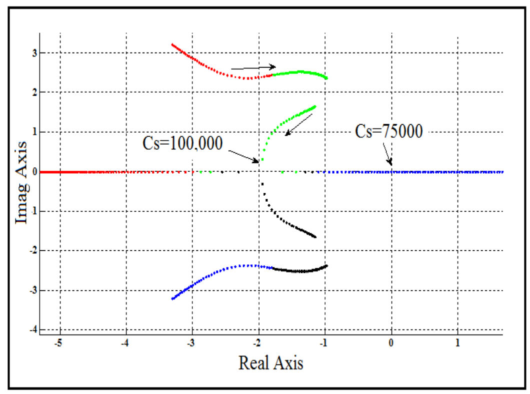

Figure 6. The effect of C_2 variation on directional stability

B. The effects of the cornering stiffness of the tractor rear axle

In order to perform the eigenvalue analysis, the C_1 and C_3 remain unchanged while the C_2 varies in the range 21000-132000 [N/deg].

In figure(6) the root locus has been plotted. Clearly, the dynamic behavior of the system is significantly affected by varying the cornering characteristics of the tractor rear wheels, unlike what is observed in tractor front wheels. It is obvious that a 40% reduction in rear cornering stiffness may lead to an unstable behavior.

To confirm the above prediction, the pervious lane change maneuver is again simulated. However, in this case the heavily braking torque applied to tractor rear axles. The AHV motion is depicted in figure(7). It is shown that the jackknifing has occurred which is incredibly difficult for the driver to control the vehicle.

It can be concluded that exceeding braking torque on tractor rear wheels may increase the tendency to jackknife.

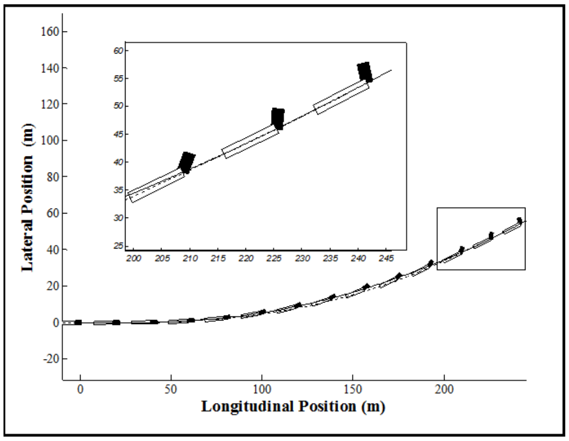

Figure 7. The animated vehicle motion in lane change manuver

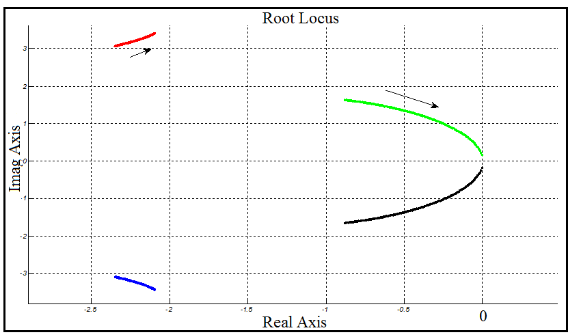

Figure 8. The effect of C_3 variation on directional stability

C. *The effects of the cornering stiffness of the trailer rear axle*

In parallel with above subsections, the trajectories of the system eigenvalues are plotted as function of C_3. The term C_3 is reduced from 132000[N/deg] to 21000 [N/deg] based on the mentioned facts.

As shown in figure (8), with decrease of C_3, the amount of the real part of the dominant eigenvalue is considerably reduced. However, the system never becomes unstable. While under extreme conditions the system becomes marginally stable.

In order to confirm the mentioned prediction in real applications, the previous maneuver is simulated. In this case the heavily braking torque applied to trailer rear axles.

The vehicle motion is depicted in figure(10). The tractor path and trailer path are shown with red and blue lines, respectively. It is clear that the trailer exhibits the sway Motion.

It can be concluded that exceeding braking torque on trailer rear wheels may increase the tendency to sway motion.

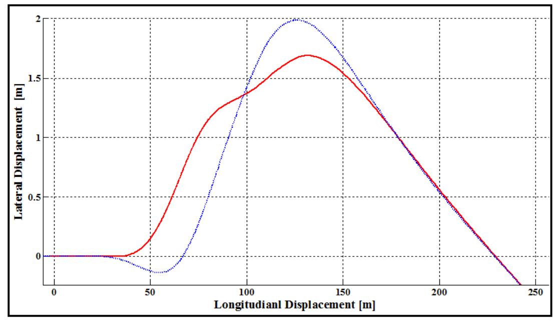

Figure 9. The vehicle path in lane change manuver

V. CONCLUSION

- The nonlinear planar model and linear planar model of articulated heavy vehicle have been developed. The linear model is used to analysis the dynamic stability of the vehicle and the nonlinear model is used to evaluate the predicted results derived from the stability analysis.
- This study showed that the directional stability of the articulated vehicle is not significantly affected by varying the cornering stiffness of the front axle of the tractor unit.
- This study showed that the reduction of the cornering stiffness of the rear axle of the tractor unit may lead the AHV to jackknife unstable motion.
- This study showed that the reduction of the cornering stiffness of the rear axle of the trailer unit may lead the trailer unit to sway motion.

APPENDIX

Notation

I_{zt} Tractor inertia moment about the yaw axis

I_{zs} Semitrailer inertia moment about the yaw axis

I_w Polar inertia moment of wheel

L_{fs} Distance between the trailer centre of gravity and coupling

L_{ft} Distance between the tractor centre of gravity and tractor

L_{rt} Distance between the tractor centre of gravity and tractor L_{rs} Distance between the trailer centre of gravity and trailer

L_{ts} Distance between adjacent trailer

L_{wt} Distance between the tractor center of gravity and coupling

m_t Total mass of the tractor unit

m_s Total mass of the trailer unit

R_w Wheel radius

T_{wt} Track width of the tractor unit

T_{ws} Track width of the trailer unit

ACKNOWLEDGMENT

This work was supported by South Tehran Branch, Islamic Azad University under Project "direction stability analysis of the articulated vehicle"

REFERENCES

[1] R. Krishna, "Directional Dynamic analysis of an articulated vehicle with articulation damper and forced steering," M.S. thesis, Dept. Mecha. Eng., Concordia Univ., Montreal, Canada, 1993.
[2] M. M. Islam, D. Xuejun and Y. He, "A closed-loop dynamic simulation-based design method for articulated heavy vehicles with active trailer steering systems," Vehicle System Dynamics, 2011, pp.1-23.
[3] F. Jindra, "Handling characteristics of tractor-trailer combinations," SAE paper 650720, 1965.
[4] R. D. Ervin, "The influence of size and weight variables on the roll stability of heavy duty trucks," SAE paper 831163,1983.
[5] P. S. Fancher, "Directional dynamics consideration for multi articulated multi-axled heavy vehicles,"SAE paper892499,1989.
[6] J.R. Ellis, "Vehicle Handling Dynamics," Page Bros, 1994.
[7] T. Kaneko, and I. Kageyama, "A study on the braking stability of articulated heavy vehicles," JSAE, Review 24, 2003, pp. 157-164.
[8] E. Dahlberg, and J.P. Wideberg, "Influence of the fifth_wheel location on heavy articulated vehicle handling," Proceedings 8th International Symposium on Heavy Vehicle Weights and Dimensions, 14th - 18th March, Johannesburg, South Africa.2004.
[9] A. Hac, D. Fulk, and H. Chen, Stability and control considerations of vehicle-trailer combination, SAE paper 2008-01-1228, 2008.
[10] Y. Feng-yan, X. Mei-zhi, and W. Chao-yi, "Stability simulation of operation for tractor semitrailer based on root-locus method," Intelligent Systems and Applications (ISA), 2nd International Workshop on, 2010.
[11] H. Dugoff, C. Fancher, and L. Segel, "An analysis of tire traction properties and their influence on vehicle dynamic performance," SAE paper 700377, 1970.
[12] J. Y. Wong, "Theory of Ground Vehicles," John Wiley & Sons, Inc., USA, 1978.